# TRANSFORMATION OF COORDINATES USING LEAST SQUARES COLLOCATION


## ABSTRACT

This paper presents three methods of transforming World Geodetic System 1984 (WGS84) coordinates to Australian Geodetic Datum 1966 (AGD66) coordinates. Traditional methods of scaling, translating and rotating are compared with the *least squares collocation* technique of *parameter estimation, filtering and interpolation* revealing some advantages of collocation.


## INTRODUCTION

The U.S. Department of *Defence Global Positioning* System (GPS) is widely used for surveying and mapping applications in Australia. GPS derived coordinates are related to the WGS84 Cartesian coordinate system $(x, y, z)_{WGS}$ whose origin is at the Earth's centre of mass. The *Z*-axis is in the direction of the Conventional Terrestrial Pole (CTP), as defined by the Bureau International de l'Heure (BIH) on the basis of coordinates adopted for the BIH stations around the world, the *X*-axis passes through the intersection of the CTP's equator and the zero meridian plane near Greenwich as defined by the BIH and the *Y*-axis is in the plane of the equator 90° east of the *X*-axis.

The WGS84 Cartesian coordinate origin also serves as the geometric centre of the WGS84 ellipsoid—whose parameters, with one minor exception, are those of the Geodetic Reference System 1980 (GRS80) ellipsoid (*Moritz*, 1980a and *Decker*, 1986). The minor axis of the WGS84 ellipsoid is coincident with the *Z*-axis and the *X-Z* and *X-Y* Cartesian planes are coincident with the zero meridian and equatorial planes of the ellipsoid respectively.

AGD66 geodetic coordinates (latitude $\phi$ longitude $\lambda$ height *h*) are related to the Australian National Spheroid (ANS) whose minor axis is parallel to the direction of the CTP as defined by the BIH and whose zero meridian plane is defined as being parallel to the BIH zero meridian plane near Greenwich. The ANS is


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an integral part of the Australian Geodetic Datum (*AGD Technical Manual,* 1986) and in this paper, spheroid and ellipsoid both refer to a geometric surface formed by an ellipse rotated about its minor axis. AGD66 Cartesian coordinates (*x, y, z*) have their origin at the centre of the ANS; with the *Z*-axis in the direction of the minor axis of the ANS, the *X*-axis passing through the intersection of the ANS equator and zero meridian plane and the *Y*-axis in the plane of the ANS equator 90° east of the *X*-axis.

Geodetic reference ellipsoids, such as the WGS84 and ANS, are computational surfaces which approximate the whole, or portions of, an irregular equipotential surface known as the *geoid,* where the geoid can be defined as (*DMA Technical Report,* 1983, p.10), "... that surface to which the oceans would conform over the entire earth if free to adjust to the combined effect of the earth's mass attraction and the centrifugal force of the earth's rotation". Since the WGS84 ellipsoid is a global approximation of the geoid and the ANS is only an approximation of the geoid for the Australasian region, it is known that the origins of the two ellipsoids do not coincide and translations between the AGD and WGS84 origins are approximately: $\delta_x = +133m$, $\delta_y = +48m$, $\delta_z = -148m$ (*DMA Technical Report,* 1987), where $x_{AGD} = x_{WGS} + \delta_x$ and similarly for *y* and *z*. Furthermore, it is often regarded that the axes of the two Cartesian systems are not exactly parallel and that a scale factor exists between vectors in both systems. It is for these reasons that it is necessary to transform WGS84 coordinates, derived from GPS measurements, into coordinates related to the AGD.

Three transformation models will be investigated; (i) a *three parameter* model involving translations only, (ii) a *seven parameter* model involving three translations, three rotations and a scale factor and (iii) least squares collocation which combines *parameter estimation, filtering* and *interpolation.* Parameters for the first two models will be derived from sets of AGD66 and WGS84 coordinates for 16 points spread across Victoria using the traditional least squares approach set out in Sections 3 and 4 respectively. The method used in performing the transformation by least squares collocation is detailed in Section 8. Comparing residuals at the 16 data points from the three transformation models indicates that the collocation approach may offer some advantages in determining the best estimates of transformed coordinates.

## RELATIONSHIPS BETWEEN CARTESIAN AND GEODETIC COORDINATES

Figure 1 below, shows the well known relationships between Cartesian coordinates (*x,y,z*) and Geodetic coordinates ($\phi$, $\lambda$, $h$) of a point *P* related to an ellipsoid whose semi-major axis is $OE = a$ and semi-minor axis is $ON = b$.

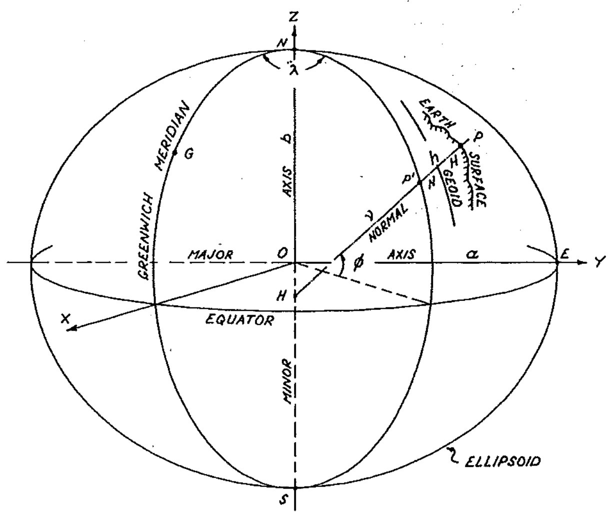


**FIGURE 1**

Referring to Figure 1, the Cartesian coordinates (*x,y,z*) of a point $P(\phi,\lambda,h)$ on an ellipsoid of semi-major axis *a* and flattening *f* may be calculated by the following formulae:

$$x = (\nu + h)\cos\phi\cos\lambda \qquad (2.1)$$

$$y = (\nu + h)\cos\phi\sin\lambda \qquad (2.2)$$

$$z = (\nu(1-e^2) + h)\sin\phi \qquad (2.3)$$

where

$\nu = HP' = \dfrac{a}{\sqrt{(1-e^2\sin^2\phi)}}$ = radius of curvature in the prime vertical plane.

$e^2 = f(2-f)$ = eccentricity squared.

$b = ON = a(1-f)$ = semi-minor axis of ellipsoid.

$OH = \nu e^2 \sin\phi$

The inverse computation of ($\phi,\lambda,h$) from (x,y,z) can be made using the following:

$$\cos\lambda = \frac{x}{r} \qquad (2.4)$$

$$\tan\phi = \frac{z + \nu e^2\sin\phi}{r} \qquad (2.5)$$

$$h = \frac{r}{\cos\phi} - \nu \qquad (2.6)$$

where

$$r = \sqrt{(x^2 + y^2)}$$

A derivation of these classical formulae above, may be found in *Torge* (1980, pp.47-52).

Note: In equation (2.5), functions of the latitude appear on both sides of the equation which requires an iterative solution for $\phi$. A first approximation for the latitude may be obtained from $r\tan\phi = z$. Convergence will be rapid since $h << \nu$.

## THREE PARAMETER TRANSFORMATION MODEL

Figure 2 shows the *3-parameter* transformation model where the two parallel Cartesian coordinate systems; *XYZ* with origin $O_1$ and *UVW* with origin $O_2$, are linked by the vector $\Delta$ whose components are the three translations $\delta_x$, $\delta_y$, $\delta_z$.

For $n$ points common to both systems, the vector equation for point $P_i$ is:

$$\mathbf{a}_i = \mathbf{b}_i + \Delta + \mathbf{v}_i \tag{3.1}$$

where

**a** and **b** are position vectors, $\Delta$ is a vector of translations and **v** is a vector of residuals.

The least squares estimate of the *three parameters* in $\Delta$, if all $3n$ coordinate pairs are considered to be of equal precision is

$$\Delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} = \begin{bmatrix} \dfrac{\sum_{i=1}^{n}(x_i - u_i)}{n} \\ \dfrac{\sum_{i=1}^{n}(y_i - v_i)}{n} \\ \dfrac{\sum_{i=1}^{n}(z_i - w_i)}{n} \end{bmatrix} = \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} - \begin{bmatrix} u_g \\ v_g \\ w_g \end{bmatrix} = \mathbf{g}_{xyz} - \mathbf{g}_{uvw} \tag{3.2}$$

where

$\mathbf{g}_{xyz}$ and $\mathbf{g}_{uvw}$ are position vectors of the centroid in both systems.

The 3-parameter transformation model assumes that the *XYZ* and *UVW* coordinate axes are parallel and no scale factor exists between vectors in both systems. This model will be used as the basis for systematic trend removal in the collocation process discussed in later sections.

## SEVEN PARAMETER TRANSFORMATION MODEL

Figure 3 shows the 7-parameter (Bursa-Wolf) transformation model (Krakiwsky and Thomson, 1974)

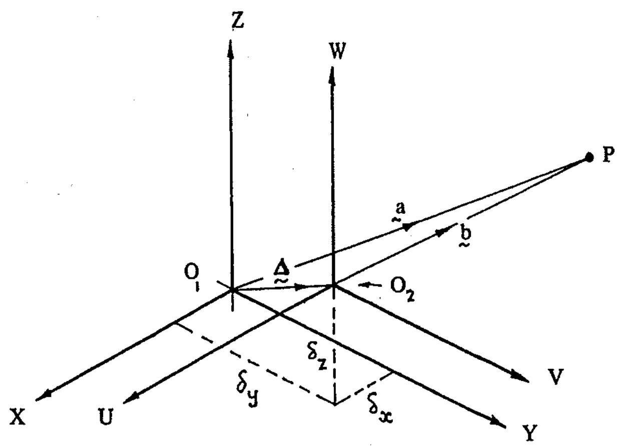


**FIGURE 2**

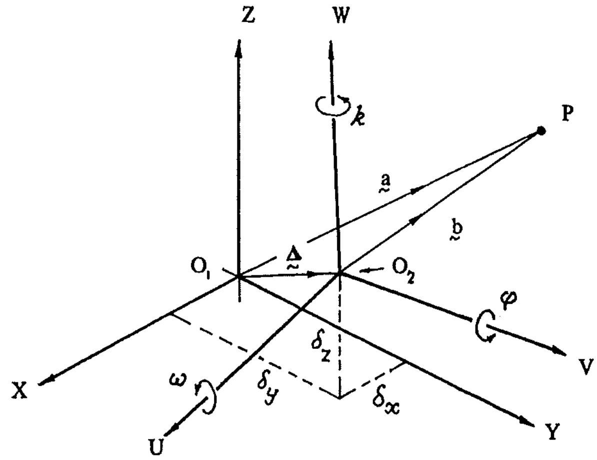


**FIGURE 3**

where the *UVW* system is *scaled, translated* and *rotated* with respect to the *XYZ* system. Small rotations ($\omega$, $\varphi$, $\kappa$) around the (*U, V, W*) axes respectively, are considered positive anti-clockwise when viewed from the positive end of the axis looking towards the origin. The product of three consecutive rotations around the axes can be expressed in rotation matrices (*Harvey*, 1986) as:

$$\mathbf{R} = \mathbf{R}_\kappa \mathbf{R}_\varphi \mathbf{R}_\omega = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \tag{4.1}$$

Since the rotations are always small, R can be approximated by

$$\mathbf{R} \approx \begin{bmatrix} 1 & \kappa & -\varphi \\ -\kappa & 1 & \omega \\ \varphi & -\omega & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \kappa & -\varphi \\ -\kappa & 0 & \omega \\ \varphi & -\omega & 0 \end{bmatrix} = \mathbf{I} + \delta\mathbf{R} \tag{4.2}$$

In a similar way to the 3-parameter transformation, but including a scale factor $\lambda$ and the rotation matrix **R**, the vector equation for $P_i$ for $n$ points common to both systems is

$$\mathbf{a}_i = \lambda \mathbf{R} \mathbf{b}_i + \Delta + \mathbf{v}_i \qquad (4.3)$$

Letting the scale factor $\lambda = 1 + \delta\lambda$ and $\mathbf{R} = \mathbf{I} + \delta\mathbf{R}$, equation (4.3) becomes

$$\mathbf{a}_i = (1 + \delta\lambda)(\mathbf{I} + \lambda\mathbf{R})\mathbf{b}_i + \Delta + \mathbf{v}_i \qquad (4.4)$$

and since $\delta\lambda$ and $\delta\mathbf{R}$ are small and the product $\delta\lambda\, \delta\mathbf{R} \approx \mathbf{0}$, equation (4.4) becomes

$$\mathbf{a}_i = \delta\mathbf{R}\mathbf{b}_i + \delta\lambda\mathbf{b}_i + \Delta + \mathbf{b}_i + \mathbf{v}_i \qquad (4.5)$$

Rearranging (4.5), each common point gives rise to an equation of the following form:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & -w & v & u & 1 & 0 & 0 \\ w & 0 & -u & v & 0 & 1 & 0 \\ -v & u & 0 & w & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega \\ \varphi \\ \kappa \\ \delta\lambda \\ \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} = \begin{bmatrix} x-u \\ y-v \\ z-w \end{bmatrix} \qquad (4.6)$$

For the $n$ points common to both systems, equations (4.6) can be represented as

$$\mathbf{v} + \mathbf{B}\Delta = \mathbf{f} \qquad (4.7)$$

and the least squares estimate of the *seven parameters* in $\Delta$, if all $3n$ coordinate pairs are considered to be of equal precision, is (*Mikhail,* 1976, section 7.3)

$$\Delta = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{f} = [\omega\ \varphi\ \kappa\ \delta\lambda\ \delta_x\ \delta_y\ \delta_z]^T \qquad (4.8)$$

The 7-parameter transformation model is the commonly accepted standard for transforming GPS derived WGS84 coordinates to AGD66 coordinates.

## THE TRANSFORMATION DATA

The transformation data were derived from a high precision GPS network covering Victoria and New South Wales, jointly conducted by the Land Information Centre, Bathurst, N.S.W. and Survey & Mapping Victoria. The part of the network covering Victoria is shown in Figure 4 and consists of 33 stations connected by 146 GPS vectors. A 3-Dimensional unconstrained adjustment of the data relating to these 33 stations (using program GeoLab™) yielded WGS84 coordinates of 32 stations; Kosciusko being held fixed at known WGS84 values. Sixteen of these stations, indicated by a $\Delta$ on the diagram, also have known AGD66 values of latitude

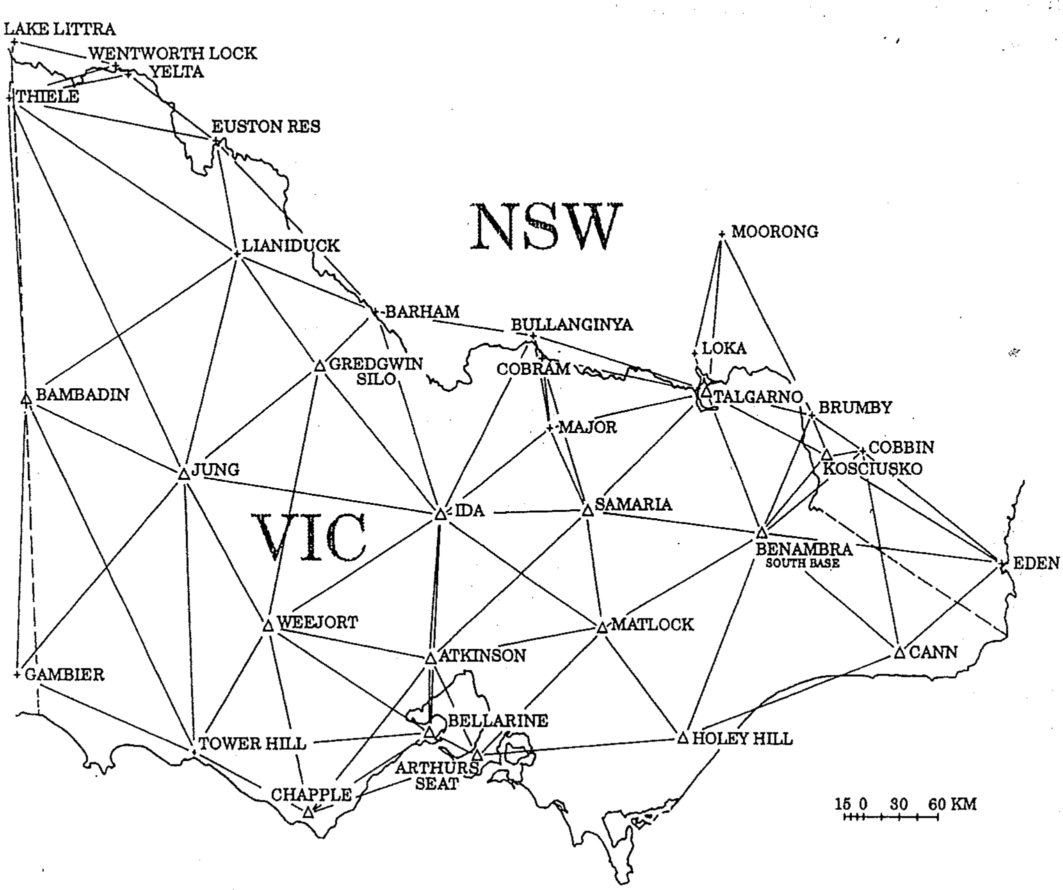


**FIGURE 4**

($\phi$), longitude ($\lambda$) and Australian Height Datum (AHD) heights ($H$ = height above the geoid). Geoid-spheroid separations ($N$) for the WGS84 ellipsoid were computed at all stations in the network using the *Ohio State University* Spherical Harmonic Gravity Field Model OSU91A (*Rapp* et al, 1991) and transformed to ANS N-values using the three parameter transformation model mentioned in Section 1 (*DMA Technical Report*, 1987). These values were then used to calculate spheroidal heights, $h_{ANS} = H_{AHD} + N_{ANS}$ at the 16 common stations. A summary of the relevant data is contained in Appendix A.

## THREE PARAMETER TRANSFORMATION RESULTS

For the 16 common stations in the GPS network, the data given in Appendix A were converted to Cartesian coordinates using equations 2.1 to 2.3 and the translation vector $\Delta$ calculated from the position vectors of the centroid in both systems as given by equations (3.2).

$$\Delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} = g_{AGD66} - g_{WGS84} = \begin{bmatrix} -4172510.928 \\ +2897331.422 \\ -3839125.664 \end{bmatrix} - \begin{bmatrix} -4172643.518 \\ +2897284.264 \\ -3838978.430 \end{bmatrix} = \begin{bmatrix} +132.590 \\ +47.158 \\ -147.234 \end{bmatrix}$$

Residuals $\mathbf{v}$ at the common stations, calculated from equation (6.1), are listed in Table 1.

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \mathbf{a}_{AGD66} - \mathbf{b}_{WGS84} - \Delta = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{AGD66} - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{WGS84} - \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} \qquad (6.1)$$

**TABLE 1**
Residuals at common stations from 3-Parameter Transformation Model

| Name | v(x) | v(y) | v(z) |
|---|---|---|---|
| Arthurs Seat | -0.444 | -0.231 | 0.128 |
| Atkinson | -0.362 | -0.190 | -0.146 |
| Bambadin (PM 3) | -0.520 | -1.016 | -0.589 |
| Bellarine (GPS Ecc) | -0.575 | -0.108 | -0.199 |
| Benambra (South Base) | 0.377 | 0.538 | 0.298 |
| Cann | 0.279 | 1.239 | 0.681 |
| Chapple | -0.516 | -0.403 | 0.085 |
| Gredgwin Silo (Ecc A) | 0.261 | 0.125 | -0.487 |
| Holey Hill | -0.486 | 0.222 | 0.279 |
| Ida | -0.145 | -0.306 | -0.348 |
| Jung | -0.084 | -0.565 | -0.646 |
| Kosciusko (Pillar) | 1.157 | 0.391 | 0.969 |
| Matlock | -0.110 | 0.118 | 0.126 |
| Samaria | 0.405 | 0.224 | 0.096 |
| Talgarno | 0.690 | 0.528 | 0.128 |
| Weejort | 0.073 | -0.565 | -0.375 |

The WGS84 Cartesian coordinates of the other 17 stations in the network were transformed to AGD66 Cartesian coordinates according to equation (6.2)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{AGD66} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{WGS84} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} \qquad (6.2)$$

and converted to Australian Map Grid (AMG66) east and north coordinates (*AGD Technical Manual*, 1986). These values are shown in Table 2 together with spheroidal heights of each station.

**TABLE 2**
AMG66 coordinates and spheroidal heights of transformed stations (3-Parameter Transformation Model)

| Name | Zone | East(m) | North(m) | h(m) |
|---|---|---|---|---|
| Barham Reservoir | 54 | 240373.799 | 6053714.397 | 108.790 |
| Brumby | 55 | 602843.182 | 5988323.494 | 421.695 |
| Bullanginya | 55 | 369205.090 | 6037359.153 | 168.129 |
| Cobbin (P) | 55 | 642062.262 | 5966103.829 | 1273.199 |
| Eden Breakwater(P) | 55 | 758401.659 | 5892746.459 | 10.807 |
| Euston Reservoir | 54 | 659873.584 | 6172243.460 | 83.584 |
| Lake Littra | 54 | 500133.561 | 6245492.764 | 33.092 |
| Lianiduck (RM3 S) | 54 | 672296.474 | 6097026.170 | 96.728 |
| Loka | 55 | 505922.898 | 6030858.722 | 674.729 |
| Major (RM3 Brass) | 55 | 382912.768 | 5974833.898 | 383.445 |
| Moorong (P) | 55 | 527619.555 | 6114041.572 | 303.793 |
| MT Gambier (7022) | 54 | 478411.691 | 5811703.065 | 193.824 |
| Thiele (SA) | 54 | 490133.895 | 6206448.631 | 52.714 |
| Tower Hill (1862) | 54 | 618740.431 | 5757483.088 | 103.278 |
| Cobram (TS 72313) | 55 | 377914.275 | 6024014.079 | 150.402 |
| Wentworth Lock | 54 | 583237.532 | 6225091.830 | 35.237 |
| Yelta (SSM) | 54 | 592592.771 | 6223118.823 | 59.137 |

## SEVEN PARAMETER TRANSFORMATION RESULTS

For the 16 common stations in the GPS network, the data given in Appendix A were converted to Cartesian coordinates using equations 2.1 to 2.3. The scale factor $\lambda$, elements of the rotation matrix **R** and the vector of translations $\Delta$ were computed using equation 4.8.

| | | |
|---|---|---|
| omega ($\omega$) | = +7.811343 E-7 radians | = +0.161 seconds |
| phi ($\varphi$) | = -2.461240 E-6 radians | = -0.508 seconds |
| kappa ($\kappa$) | = +2.073098 E-7 radians | = +0.043 seconds |
| scale ($\lambda$) | = 0.999997194 (-2.81 ppm) | = +0.043 seconds |
| $\delta(x)$ | = +129.728 m | |
| $\delta(y)$ | = +57.423 m | |
| $\delta(z)$ | = -166.014 m | |

Re-ordering equation (4.5), residuals are calculated by (7.1) and are tabulated in Table 3.

$$\mathbf{v}=\begin{bmatrix}v_x\\v_y\\v_z\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}_{AGD66}-\begin{bmatrix}x\\y\\z\end{bmatrix}_{WGS84}-\delta\mathbf{R}\begin{bmatrix}x\\y\\z\end{bmatrix}_{WGS84}-\delta\lambda\begin{bmatrix}x\\y\\z\end{bmatrix}_{WGS84}-\begin{bmatrix}\delta_x\\\delta_y\\\delta_z\end{bmatrix} \quad (7.1)$$

**TABLE 3**
Residuals at common stations from 7-Parameter Transformation Model

| Name | v(x) | v(y) | v(z) |
|---|---|---|---|
| Arthurs Seat | 0.004 | -0.199 | 0.017 |
| Atkinson | -0.107 | -0.083 | -0.126 |
| Bambadin (PM 3) | -0.376 | -0.076 | 0.372 |
| Bellarine (GPS Ecc) | -0.162 | 0.001 | -0.222 |
| Benambra (South Base) | -0.021 | 0.018 | -0.114 |
| Cann | -0.024 | 0.417 | -0.043 |
| Chapple | 0.241 | -0.068 | 0.201 |
| Gredgwin Silo (Ecc A) | -0.019 | 0.510 | -0.009 |
| Holey Hill | -0.328 | -0.167 | -0.169 |
| Ida | -0.226 | -0.178 | -0.208 |
| Jung | 0.049 | 0.069 | -0.027 |
| Kosciusko (Pillar) | 0.476 | -0.250 | 0.525 |
| Matlock | -0.114 | -0.091 | -0.104 |
| Samaria | 0.142 | 0.059 | -0.009 |
| Talgarno | 0.006 | 0.151 | -0.052 |
| Weejort | 0.459 | -0.114 | -0.033 |

The WGS84 Cartesian coordinates of the other 17 stations in the network were transformed to AGD66 Cartesian coordinates according to equation (7.2).

$$\begin{bmatrix}x\\y\\z\end{bmatrix}_{AGD66}=\lambda\mathbf{R}\begin{bmatrix}x\\y\\z\end{bmatrix}_{WGS84}+\begin{bmatrix}\delta_x\\\delta_y\\\delta_z\end{bmatrix} \quad (7.2)$$

and converted to AMG66 east and north coordinates. These coordinates together with spheroidal heights of each station are shown in Table 4.

## THE LEAST SQUARES COLLOCATION MODEL

*Mikhail* (1976, p.394) describes collocation as "a general least squares technique combining classical adjustment with *interpolation* and *filtering*...", where interpolation is the estimation of quantities at locations where no observational data are given and filtering is the estimation of these quantities taking into account the random measuring errors assumed to have occurred at the data points. *Mikhail* (1976) and *Krakiwsky* (1975) both demonstrate that least squares parameter estimation (adjustment) and least squares interpolation and filtering are special cases of collocation and *Moritz* (1980b, p.132) provides ample mathematical proof that, as is the case with traditional least squares, "... collocation is optimal in the sense that it gives the most accurate results that are obtainable on the basis of the available data".

To develop the collocation equations, consider equation (4.7) where the transformation model for $n$ observations of $u$ parameters can be represented as

$$\mathbf{v}_{(n,1)}+\mathbf{B}_{(n,u)}\Delta_{(u,1)}=\mathbf{f}_{(n,1)} \quad (4.7)$$

and suppose that the residuals **v** are decomposed into a correlated *signal* component **s** and a random *noise* component **n**. The noise in the model is simply the random measuring errors and the signal can be described as that component of the model which reflects the inability of the $u$ selected parameters $\Delta$ to accurately describe the physical relationships. Furthermore, the signal component can be subdivided into signals at the $n$ *observation* points **t** and signals at the $m$ *computation* points **u**. These three random vectors can be combined as

$$\dot{\mathbf{v}}=\begin{bmatrix}\mathbf{s}\\\mathbf{n}\end{bmatrix}\quad\text{where}\quad\mathbf{s}=\begin{bmatrix}\mathbf{u}\\\mathbf{t}\end{bmatrix}\quad\text{giving}\quad\dot{\mathbf{v}}=\begin{bmatrix}\mathbf{u}\\\mathbf{t}\\\mathbf{n}\end{bmatrix} \quad (8.1)$$

and so equation (4.7) becomes

$$\begin{bmatrix}\mathbf{0}_{(m,m)} & \mathbf{I}_{(n,n)} & \mathbf{I}_{(n,n)}\end{bmatrix}\begin{bmatrix}\mathbf{u}_{(m,1)}\\\mathbf{t}_{(n,1)}\\\mathbf{n}_{(n,1)}\end{bmatrix}+\mathbf{B}_{(n,u)}\Delta_{(u,1)}=\mathbf{f}_{(n,1)} \quad (8.2)$$

or

$$\dot{\mathbf{A}}\dot{\mathbf{v}}+\mathbf{B}\Delta=\mathbf{f} \quad (8.3)$$

Assuming that no correlation exists between signal and noise, the a-priori variance-covariance matrix associated with the random quantities in equation (8.3) is

$$\mathbf{C}=\begin{bmatrix}\mathbf{C}_{ss} & 0\\0 & \mathbf{C}_{nn}\end{bmatrix}\quad\text{and}\quad\mathbf{C}_{ss}=\begin{bmatrix}\mathbf{C}_{uu} & \mathbf{C}_{ut}\\\mathbf{C}_{ut}^T & \mathbf{C}_{tt}\end{bmatrix}\quad\text{hence}\quad\mathbf{C}=\begin{bmatrix}\mathbf{C}_{uu} & \mathbf{C}_{ut} & 0\\\mathbf{C}_{ut}^T & \mathbf{C}_{tt} & 0\\0 & 0 & \mathbf{C}_{nn}\end{bmatrix} \quad (8.4)$$

**TABLE 4**
AMG66 coordinates and spheroidal heights of transformed stations (7-Parameter Transformation Model)

| Name | Zone | East(m) | North(m) | h(m) |
|---|---|---|---|---|
| Barham Reservoir | 54 | 240373.766 | 6053713.718 | 108.592 |
| Brumby | 55 | 602842.267 | 5988323.631 | 421.214 |
| Bullanginya | 55 | 369204.732 | 6037358.750 | 167.826 |
| Cobbin (P) | 55 | 642061.277 | 5966104.098 | 1272.707 |
| Eden Breakwater(P) | 55 | 758400.480 | 5892747.118 | 10.297 |
| Euston Reservoir | 54 | 659873.637 | 6172242.226 | 83.324 |
| Lake Littra | 54 | 500133.918 | 6245491.030 | 32.871 |
| Lianiduck (RM3 S) | 54 | 672296.630 | 6097025.168 | 96.579 |
| Loka | 55 | 505922.175 | 6030858.581 | 674.287 |
| Major (RM3 Brass) | 55 | 382912.484 | 5974833.694 | 383.231 |
| Moorong (P) | 55 | 527618.631 | 6114041.239 | 303.187 |
| MT Gambier (7022) | 54 | 478412.920 | 5811702.516 | 194.317 |
| Thiele (SA) | 54 | 490134.354 | 6206446.984 | 52.568 |
| Tower Hill (1862) | 54 | 618741.355 | 5757482.951 | 103.712 |
| Cobram (TS 72313) | 55 | 377913.916 | 6024013.729 | 150.112 |
| Wentworth Lock | 54 | 583237.698 | 6225090.310 | 34.967 |
| Yelta (SSM) | 54 | 592592.915 | 6223117.325 | 58.861 |

where

$C_{uu}$ is the (m,m) variance-covariance matrix for the signals at the computation points,

$C_{tt}$ is the (n,n) variance-covariance matrix for the signals at the observation points,

$C_{ut}$ is the (m,n) covariance matrix between the signals at the computation and observation points, and

$C_{nn}$ is the (n,n) variance-covariance matrix of the noise.

Applying the least squares principle to equation (8.3) in the manner of *Mikhail* (1976, pp.419-420) leads to the best estimates for the parameters, signals and noise as

$$\Delta_{(u,1)} = (B^T D^{-1} B)^{-1} (B^T D^{-1} f) \quad (8.5)$$

$$\mathbf{u}_{(m,1)} = C_{ut} D^{-1} (f - B\Delta) \quad (8.6)$$

$$\mathbf{t}_{(n,1)} = C_{tt} D^{-1} (f - B\Delta) \quad (8.7)$$

$$\mathbf{n}_{(n,1)} = C_{nn} D^{-1} (f - B\Delta) \quad (8.8)$$

where

$$D_{(n,n)} = (C_{tt} + C_{nn}) \quad (8.9)$$

The mathematical model described above (equations 8.1 to 8.4) and its solution (equations 8.5 to 8.8) is known as *Collocation with Parameters* and allows the simultaneous estimation of: (i) the vector of parameters $\Delta$; (ii) the vector of signals **u** at the computation points, (known as *interpolation*); (iii) the vector of signals **t** at the observation points, (known as filtering); as well as (iv) the vector of random noise **n** at the observation points. Those familiar with the traditional least squares approach will notice the strong resemblance to the collocation model and its solution. The underlying difference between the two approaches is the incorporation of the signal term **s** and the components of its variance-covariance matrix $\mathbf{C}_{ss}$ in the solution equations. In fact, it is this matrix $\mathbf{C}_{ss}$ which represents the central point in collocation and allows quantities, which are normally linked by mathematical relationships, to be described in a statistical manner, ie. by the use of variance-covariance matrices.

In describing an application of collocation *Cross* (1992, p.142) uses the example of predicting the unknown height of a point surrounded by a number of other points of known height. The location of all points are known and it is assumed that a distance dependent function is known which is capable of computing the covariance of the heights of any two points in the region. This *covariance function* enables all the elements of a height variance-covariance matrix to be computed which describes the variation of height in the region in a statistical manner. In flat areas, the heights of neighbouring points would be highly correlated (large covariances), but in highly undulating areas, points would be weakly correlated (small covariances). Solving the collocation equations, with the appropriate variance-covariance matrices, allows unknown heights of points within the region to be interpolated (or predicted). This approach contrasts with the usual method of fitting a surface to the known points and using the parameters of that surface to compute heights at other points. Determining the appropriate covariance function to compute the elements of $\mathbf{C}_{ss}$ is the central issue in collocation.

## THE COLLOCATION MODEL APPLIED TO 3-D CARTESIAN COORDINATE TRANSFORMATIONS

In the collocation model described above, the selection of the $u$ parameters in $\Delta$ depends on whether it is assumed the coordinate systems are related by translations only, or by scale, rotations and translations. In the former, $\Delta = [\delta_x\ \delta_y\ \delta_z]^T$ and $u = 3$, whilst in the latter $\Delta = [\omega\ \varphi\ \kappa\ \delta\lambda\ \delta_x\ \delta_y\ \delta_z]^T$ and $u = 7$. In the case of three parameters the coefficient matrix $\mathbf{B}_i$ for each of the $n$ points common to both systems will be equal to the identity matrix **I** and in the case of seven parameters $\mathbf{B}_i$ will take the form given in equation (4.6). In both cases, the vector **f** will contain numeric terms which are coordinate differences $(x, y, z)_{AGD66} - (x, y, z)_{WGS84}$.

In this paper, for simplicity, the collocation model will assume $\Delta = [\delta_x\ \delta_y\ \delta_z]^T$ and $u = 3$.

## THE COVARIANCE FUNCTION FOR THE MODEL

The covariance function, necessary for computing the elements of the signal variance-covariance matrices $\mathbf{C}_{tt}$ and $\mathbf{C}_{ub}$ must be determined empirically from the observational data and as a prerequisite, the systematic component of the data, modelled by $\mathbf{B}\Delta$, must be removed. This is known as *trend* removal and $\mathbf{B}\Delta$ is referred to as the trend surface. Inspection of equations (8.6 to 8.8) shows that **u**, **t** and **n** are determined from the observational data after trend removal, expressed in the equations as $(\mathbf{f} - \mathbf{B}\Delta)$. Since a collocation model with $\Delta = [\delta_x\ \delta_y\ \delta_z]^T$ and $u = 3$ is assumed, then the residuals arising from the 3-parameter transformation (given in Table 1) are in fact the observational data with the trend removed and are represented by $\hat{\mathbf{v}}$ in equation (8.1). It is commonplace to assume that any correlation between these "observed" quantities $\mathbf{l} = [l_1, l_2, l_3 \dots l_n]^T$ is distance dependent and variances and covariances are calculated (*Mikhail*, 1976, p.405) as

Variance: $$C_l(0) = \frac{1}{n}\sum_{p=1}^{n} l_p^2 \qquad (10.1)$$

Covariance: $$C_l(d_k) = \frac{1}{n_k}\sum_{i<j} l_i l_j \qquad (10.2)$$

Equation (10.1) shows that the variance is computed by summing the squares of all data values $l_p$ and dividing by the number of data points $n$. Equation (10.2) shows that a covariance may be computed from the $n_k$ data pairs (or products) falling within a particular distance class interval $d_k$. The sixteen (16) data points common to both systems give rise to $(n^2 - n)/2 = 120$ data products which can be grouped into spatial distance classes $d_k$ and a covariance calculated for each distance class. Table 5 shows covariances ($cm^2$) in $x,y,z$ directions for particular distance classes and the number of data products used to calculate each covariance.

**TABLE 5**
Spatial Distance Class Covariances

| Dist Class (km) | x-cov | y-cov (cm squared) | z-cov | prods |
|---|---|---|---|---|
| 0-25 | 0.00 | 0.00 | 0.00 | 0 |
| 25-50 | 2317.87 | 227.79 | 17.17 | 2 |
| 50-75 | 1605.57 | 438.02 | -186.79 | 1 |
| 75-100 | 1243.42 | 801.60 | 966.15 | 4 |
| 100-125 | 2582.68 | 1569.98 | 707.76 | 5 |
| 125-150 | 714.88 | 1578.30 | 1236.20 | 16 |
| 150-175 | -45.84 | 264.70 | 174.94 | 8 |
| 175-200 | -1062.90 | 536.03 | 556.58 | 6 |
| 200-225 | 1190.43 | 360.77 | 691.31 | 6 |
| 225-250 | -570.16 | 1423.96 | 673.39 | 12 |
| 250-275 | 9.06 | 76.55 | 251.88 | 6 |
| 275-300 | -1092.10 | -494.17 | -408.29 | 9 |
| 300-325 | -1082.27 | -687.21 | -627.39 | 6 |
| 325-350 | -1032.65 | -38.37 | -257.17 | 4 |

Variances calculated from the data are $\sigma_x^2 = 2341.20\ cm^2$, $\sigma_y^2 = 2758.08\ cm^2$ and $\sigma_z^2 = 1849.36\ cm^2$. Figures 5, 6 and 7 show graphs of covariance versus spatial distance (solid line) where the covariance is plotted at the mid-point of the particular data class. Superimposed over these graphs is a dotted line representing a distance dependent covariance function of a Gaussian form represented by the equation

$$C(d) = ce^{-(a^2d^2)} \qquad (10.3)$$

where

$C(d)$ is the covariance between two points distance d apart,

$c$ is a constant equal to the variance and,

$a$ is a constant.

The constants c and a for these three covariance functions were derived from a weighted least squares "best fit" solution of the positive covariances shown in Table 5, where weights were assigned according to the number of data pairs used to calculate the covariance.

The three covariance functions shown as dotted lines in Figures 5, 6 and 7 are

$$C_x(d) = 2438\, e^{-\left(\frac{d}{132}\right)^2} \qquad (10.4)$$

$$C_y(d) = 1792\, e^{-\left(\frac{d}{194}\right)^2} \qquad (10.5)$$

$$C_z(d) = 1047\, e^{-\left(\frac{d}{257}\right)^2} \qquad (10.6)$$

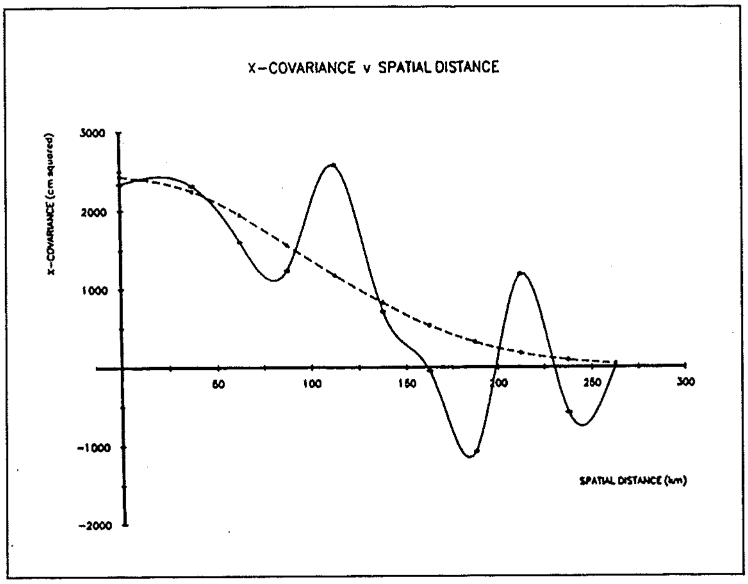


**FIGURE 5**

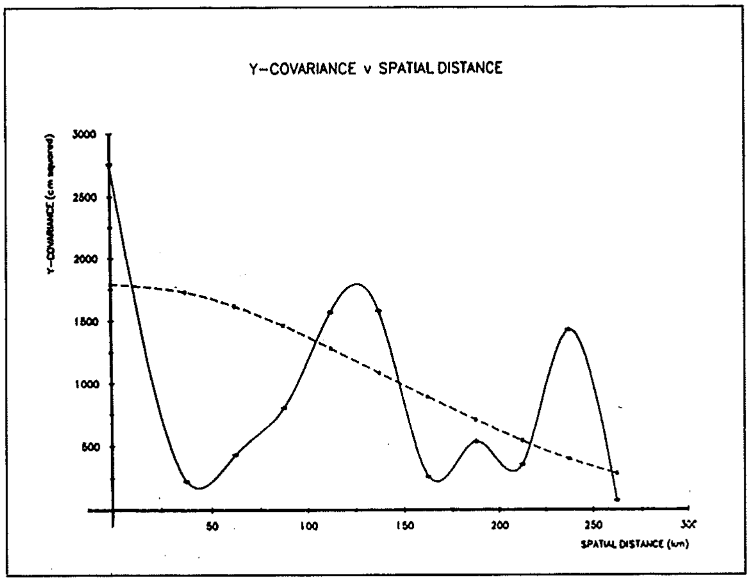


**FIGURE 6**

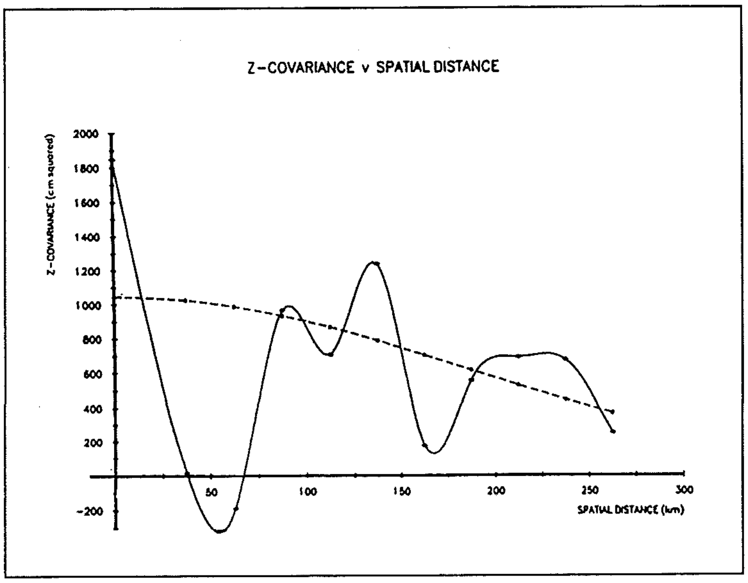


**FIGURE 7**

These covariance functions were used to compute the elements of matrices $\mathbf{C}_{tt}$ and $\mathbf{C}_{ut}$.

The noise **n** in the model is assumed to be random and furthermore, it is assumed that no correlation exists between signals **s** and noise **n**. *Mikhail* (1976, pp.395-399) shows that the variances of the noise can be calculated from the relationship

$$C_l(0) = C_s(0) + C_n(0) \qquad (10.7)$$

where

$C_l\ C_s\ C_n$ are covariances of observations, signals and noise respectively,

$C(0)$ is the covariance between two points distance $d = 0$ apart; or in other words, the variance.

Using this relationship, the elements of the diagonal noise variance matrix $\mathbf{C}_{nn}$ were calculated from the observations and the covariance functions as

$$\sigma_x^2 = |2341 - 2438| = 97\ cm^2 \qquad \sigma_x = \pm 0.098\ m$$

$$\sigma_y^2 = 2758 - 1792 = 966\ cm^2 \qquad \sigma_y = \pm 0.311\ m$$

$$\sigma_z^2 = 1849 - 1047 = 802\ cm^2 \qquad \sigma_z = \pm 0.283\ m$$

## COLLOCATION RESULTS

For the 16 common stations in the GPS network the parameters Δ were computed using equation (8.5) as

$$\Delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} = \begin{bmatrix} +132.622 \\ +47.163 \\ -147.205 \end{bmatrix} \text{ metres}$$

The small differences between these values and those obtained from the 3-parameter model, which assumes "measurements" of equal precision, are due to the fact that the variance-covariance matrix **D** is included in the collocation solution. The matrix **D** reflects the varying precision between the "measurements" in the collocation model.

The signals **u** and **t** and the noise **n** were computed using equations (8.6), (8.7) and (8.8) and are given in Tables 6, 7 and 8.

**TABLE 6**
Signals **u** at the 17 computation points

| Name | u(x) | u(y) | u(z) |
|---|---|---|---|
| Barham Reservoir | 0.198 | 0.064 | -0.362 |
| Brumby | 1.082 | 0.432 | 0.443 |
| Bullanginya | 0.206 | 0.169 | -0.134 |
| Cobbin (P) | 1.082 | 0.495 | 0.499 |
| Eden Breakwater(P) | 0.315 | 0.572 | 0.459 |
| Euston Reservoir | 0.043 | -0.042 | -0.289 |
| Lake Littra | -0.016 | -0.121 | -0.159 |
| Lianiduck (RM3 S) | 0.120 | -0.097 | -0.411 |
| Loka | 0.586 | 0.319 | 0.209 |
| Major (RM3 Brass) | 0.317 | 0.140 | -0.093 |
| Moorong (P) | 0.267 | 0.208 | 0.174 |
| MT Gambier (7022) | -0.054 | -0.359 | -0.256 |
| Thiele (SA) | -0.045 | -0.197 | -0.207 |
| Tower Hill (1862) | -0.094 | -0.379 | -0.192 |
| Cobram (TS 72313) | 0.251 | 0.173 | -0.114 |
| Wentworth Lock | -0.012 | -0.099 | -0.207 |
| Yelta (SSM) | -0.009 | -0.092 | -0.211 |

**TABLE 7**
Signals **t** at the 16 observation (common) points

| Name | t(x) | t(y) | t(z) |
|---|---|---|---|
| Arthurs Seat | -0.499 | -0.171 | -0.030 |
| Atkinson | -0.424 | -0.221 | -0.155 |
| Bambadin (PM 3) | -0.533 | -0.678 | -0.448 |
| Bellarine (GPS Ecc) | -0.549 | -0.231 | -0.096 |
| Benambra (South Base) | 0.376 | 0.540 | 0.404 |
| Cann | 0.242 | 0.777 | 0.482 |
| Chapple | -0.531 | -0.312 | -0.083 |
| Gredgwin Silo (Ecc A) | 0.212 | -0.077 | -0.449 |
| Holey Hill | -0.502 | 0.254 | 0.231 |
| Ida | -0.147 | -0.118 | -0.272 |
| Jung | -0.107 | -0.517 | -0.516 |
| Kosciusko (Pillar) | 1.066 | 0.496 | 0.474 |
| Matlock | -0.131 | 0.122 | 0.093 |
| Samaria | 0.337 | 0.162 | 0.023 |
| Talgarno | 0.658 | 0.348 | 0.235 |
| Weejort | 0.022 | -0.446 | -0.350 |

**TABLE 8**
Noise **n** at the 16 observation (common) points

| Name | n(x) | n(y) | n(z) |
|---|---|---|---|
| Arthurs Seat | 0.023 | -0.065 | 0.130 |
| Atkinson | 0.030 | 0.027 | -0.019 |
| Bambadin (PM 3) | -0.019 | -0.342 | -0.169 |
| Bellarine (GPS Ecc) | -0.058 | 0.119 | -0.132 |
| Benambra (South Base) | -0.031 | -0.006 | -0.135 |
| Cann | 0.005 | 0.458 | 0.171 |
| Chapple | -0.017 | -0.096 | 0.139 |
| Gredgwin Silo (Ecc A) | 0.017 | 0.198 | -0.067 |
| Holey Hill | -0.016 | -0.037 | 0.020 |
| Ida | -0.030 | -0.192 | -0.104 |
| Jung | -0.009 | -0.053 | -0.159 |
| Kosciusko (Pillar) | 0.060 | -0.110 | 0.466 |
| Matlock | -0.011 | -0.009 | 0.005 |
| Samaria | 0.037 | 0.057 | 0.045 |
| Talgarno | 0.000 | 0.176 | -0.136 |
| Weejort | 0.019 | -0.124 | -0.053 |

The WGS84 Cartesian coordinates of the other 17 stations in the network were transformed to AGD66 Cartesian coordinates according to equation (11.1)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{AGD66} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{WGS84} + \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} \qquad (11.1)$$

and converted to AMG66 east and north coordinates. These values are shown in Table 9 together with the spheroidal heights of each station.

**TABLE 9**
AMG66 coordinates and spheroidal heights of transformed stations (Collocation Transformation Model)

| Name | Zone | East(m) | North(m) | h(m) |
|---|---|---|---|---|
| Barham Reservoir | 54 | 240373.618 | 6053714.034 | 108.865 |
| Brumby | 55 | 602842.225 | 5988323.462 | 420.840 |
| Bullanginya | 55 | 369204.813 | 6037359.004 | 168.111 |
| Cobbin (P) | 55 | 642061.256 | 5966103.858 | 1272.330 |
| Eden Breakwater(P) | 55 | 758400.999 | 5892746.861 | 10.503 |
| Euston Reservoir | 54 | 659873.564 | 6172243.198 | 83.664 |
| Lake Littra | 54 | 500133.641 | 6245492.607 | 33.094 |
| Lianiduck (RM3 S) | 54 | 672296.449 | 6097025.754 | 96.805 |
| Loka | 55 | 505922.291 | 6030858.716 | 674.312 |
| Major (RM3 Brass) | 55 | 382912.456 | 5974833.718 | 383.317 |
| Moorong (P) | 55 | 527619.217 | 6114041.660 | 303.564 |
| MT Gambier (7022) | 54 | 478411.981 | 5811702.257 | 193.800 |
| Thiele (SA) | 54 | 490134.054 | 6206448.422 | 52.722 |
| Tower Hill (1862) | 54 | 618740.763 | 5757482.845 | 103.238 |
| Cobram (TS 72313) | 55 | 377913.971 | 6024013.927 | 150.344 |
| Wentworth Lock | 54 | 583237.592 | 6225091.638 | 35.275 |
| Yelta (SSM) | 54 | 592592.826 | 6223118.632 | 59.179 |

## COMPARISON OF TRANSFORMATION RESULTS

Table 10 shows differences between *published* AMG66 and AHD values and the three transformation models; *3-parameter, 7-parameter* and *collocation.*

The reader may draw conclusions as to the relative worth of the three transformation models based on the differences in Table 10 (published-transformed) but a statistical analysis of the residuals **v** for the 3- and 7-parameter models and the noise n in the collocation model may be useful in quantifying the precision of the three transformation methods.

From Tables 1, 3 and 8, the standard deviations of the residuals **v** and the noise **n** are

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{bmatrix} = \begin{bmatrix}\pm 0.50\\ \pm 0.54\\ \pm 0.44\end{bmatrix}_{3-PARAM}, \begin{bmatrix}\pm 0.24\\ \pm 0.21\\ \pm 0.20\end{bmatrix}_{7-PARAM}, \begin{bmatrix}\pm 0.03\\ \pm 0.18\\ \pm 0.17\end{bmatrix}_{COLL} \quad metres$$

These standard deviations indicate that the collocation model is of comparable precision to the 7-parameter model and far better than the 3-parameter model.

The means and standard deviations of the differences in Table 10 are tabulated below

$$\begin{bmatrix}\bar{E}\pm\sigma_E\\ \bar{N}\pm\sigma_N\\ \bar{H}\pm\sigma_H\end{bmatrix} = \begin{bmatrix}-0.62\pm 0.40\\ -0.23\pm 0.53\\ 0.16\pm 0.26\end{bmatrix}_{3-PARAM}, \begin{bmatrix}-0.21\pm 0.34\\ 0.04\pm 0.20\\ 0.46\pm 0.31\end{bmatrix}_{7-PARAM}, \begin{bmatrix}-0.25\pm 0.28\\ -0.17\pm 0.42\\ 0.44\pm 0.26\end{bmatrix}_{COLL} \quad metres$$

These values indicate that collocation is able to predict coordinates and heights to accuracies comparable with the 7-parameter transformation and generally better than the 3-parameter model.

## DISCUSSION

The theoretical foundations of collocation are described in texts such as *Moritz* (1980b), *Mikhail* (1976) and *Krakiwsky* (1975) but little mention is made of applications to coordinate transformations. *Moritz* (1972, pp.51-66) and *Cross* (1992, pp.150-151) describe how collocation can be applied to coordinate transformations and *Ruffhead* (1987) uses the technique to transform the grid coordinates of 15 Ordnance Survey Stations in England and Wales from the OSGB36 system to the OSGB70(SN) system. *Ruffhead* describes his results as remarkable when compared to other two-dimensional transformation models but Vincenty (1987), who re-worked Mr Ruffhead's data, comments that collocation is complicated and that a 5th. order polynomial transformation gives better results. *Vincenty,* in his final comments on the plane transformations he investigated, notes that no consideration had been given to internal distortions existing within the data. This is a crucial point and it should be emphasised that collocation *does* provide a means by which distortions can be modelled statistically, provided that *sufficient* data is available to determine the necessary covariance functions. Determining the *correct*

**TABLE 10**
Differences between published AMG66 and AHD values and transformation values

| Name | Zone | East(m) | North(m) | H(AHD) |
|---|---|---|---|---|
| Brumby | 55 | 602842.432 | 5988323.738 | 412.11 |
| (3-parameter) | | -0.750 | 0.244 | 0.07 |
| (7-parameter) | | 0.165 | 0.107 | 0.55 |
| (collocation) | | 0.207 | 0.276 | 0.92 |
| Bullanginya | 55 | 369204.524 | 6037359.064 | 162.21 |
| | | -0.566 | -0.089 | 0.21 |
| | | -0.208 | 0.314 | 0.51 |
| | | -0.289 | 0.060 | 0.23 |
| Cobbin (P) | 55 | 642061.369 | 5966104.178 | 1262.39 |
| | | -0.893 | 0.349 | -0.21 |
| | | 0.092 | 0.080 | 0.28 |
| | | -0.464 | 0.068 | 0.12 |
| Eden Breakwater(P) | 55 | 758400.535 | 5892746.929 | 5.20 |
| | | -1.124 | 0.470 | -0.19 |
| | | 0.055 | -0.189 | 0.32 |
| | | -0.464 | 0.068 | 0.12 |
| Lianiduck (RM3 S) | 54 | 672295.777 | 6097025.081 | 91.86 |
| | | -0.697 | -1.089 | 0.57 |
| | | -0.853 | -0.087 | 0.72 |
| | | -0.672 | -0.673 | 0.50 |
| Loka | 55 | 505921.984 | 6030858.580 | 667.86 |
| | | -0.914 | -0.142 | ? |
| | | -0.191 | -0.001 | ? |
| | | -0.307 | -0.316 | ? |
| Major (RM3 Brass) | 55 | 382912.181 | 5974833.706 | 376.45 |
| | | -0.587 | -0.192 | 0.08 |
| | | -0.303 | 0.012 | 0.29 |
| | | -0.275 | -0.012 | 0.20 |
| Moorong (P) | 55 | 527618.889 | 6114041.055 | 297.72 |
| | | -0.666 | -0.517 | 0.42 |
| | | 0.258 | -0.184 | 1.02 |
| | | -0.328 | -0.605 | 0.65 |
| Tower Hill (1862) | 54 | 618740.869 | 5757482.756 | 95.5 |
| | | 0.438 | -0.332 | 0.3 |
| | | -0.486 | -0.195 | -0.1 |
| | | 0.106 | -0.089 | 0.4 |
| Cobram (TS 72313) | 55 | 377913.656 | 6024014.011 | 144.34 |
| | | -0.619 | -0.068 | 0.24 |
| | | -0.260 | 0.282 | 0.53 |
| | | -0.315 | 0.084 | 0.30 |
| Yelta (SSM) | 54 | 595592.332 | 6223117.652 | ? |
| (3-parameter) | | -0.439 | -1.171 | ? |
| (7-parameter) | | -0.583 | 0.327 | ? |
| (collocation) | | -0.494 | -0.980 | ? |

covariance functions is the heart of collocation and as *Vincenty* notes: "Mathematics alone cannot perform miracles".

The AGD66 coordinate data used in this exercise is known to be distorted and various analyses by Survey & Mapping Victoria have shown that these distortions vary in magnitude and direction across the State. The covariance functions shown in Figures 5, 6 and 7, attempt to model these distortions but it is obvious from the distribution of the common points (Figure 4) and the very small number of products involved in the empirical covariances (Table 5) that the form of these functions are open to question. Nevertheless, there is some justification for believing that distance dependent correlation exists within the data and this is verified by the general reduction in the magnitude of the residuals at the common points, particularly in the x-direction, when compared with the 7-parameter transformation.

It should also be noted that covariance functions tend to lose their meaning for distances exceeding certain limits and a variable known as the *correlation length* ($\xi$) is used as a measure, where $\xi$ is that distance for which $C(\xi) = 0.5\ c$ in equation (10.3). For the three covariance functions used (given in equations 10.4, 10.5 and 10.6) the correlation lengths are $\xi$ = 110 km, 162 km and 214 km respectively. It is generally accepted in the literature that covariances may be ignored at distances greater than $1.5\xi$, but in this investigation, covariances have been evaluated for all distances and it may be that results would change if distance limits were imposed on the covariance functions. This aspect will be the subject of future investigations in coordinate transformations.

Once suitable covariance functions have been developed for a particular region, transformation between coordinate datums is a relatively simple process involving the formation of the matrix $\mathbf{C}_{ut}$ only. Equation (8.6) is used to solve for the signals **u** at the desired computation points, with the matrices **D**, **B**, $\Delta$ and **f** known constants for the particular region and transformed coordinates obtained by equation (11.1). This process would be entirely transparent to a computer user and in a similar way to the 3- and 7-parameter transformations, no further computation of transformation "parameters" is required.

In any least squares solution, a-posteriori precision estimation of parameters and residuals is possible and in collocation, estimates of precision for the parameters $\Delta$, the signals **u** and **t** and the noise **n** can be made (*Mikhail,* 1976, pp.421-423). In this paper, no precision estimations have been made and it may be that an analysis of this type could provide valuable information when choosing one transformation method in preference to others.

## ACKNOWLEDGMENTS

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## APPENDIX A

Table of data for GPS network shown in Figure 4.

Notes: 1. For the 16 common stations, the first values for each station are published AGD66 geodetic coordinates and the second values are WGS84 coordinates from an unconstrained GeoLab™ adjustment (Kosciusko held fixed). For the AGD66 values: spheroidal height $h = H + N$ where $H$ are AHD levelled heights and $N$ are geoid-spheroid separation values computed using OSU91A geoid model and transformed to the ANS (see Sec. 5).

2. For the 17 other stations in the network, the values are WGS84 geodetic coordinates from the GeoLab™ adjustment and geoid-spheroid separation values related to the ANS computed using the OSU91A geoid model (see Sec. 5).

3. Spheroids: ANS a = 6378160.0 m f = 1/298.25
WGS84 a = 6378137.0 m f = 1/298.257223563

### COMMON STATIONS

| Name | Lat.(DMS) | Long.(DMS) | h(m) | H(m) (AHD) | N(m) (ANS) |
|---|---|---|---|---|---|
| ARTHURS SEAT | -38 21 18.5350 | 144 56 57.8281 | 326.798 | 318.628 | 8.17 |
| | -38 21 13.14053 | 144 57 2.53573 | 321.98791 | | |
| ATKINSON | -37 45 32.2061 | 144 40 53.6989 | 149.021 | 140.361 | 8.66 |
| | -37 45 26.80477 | 144 40 58.38740 | 145.38617 | | |
| BAMBADIN (PM 3) | -36 7 9.4677 | 140 58 36.7367 | 161.005 | 154.455 | 6.55 |
| | -36 7 4.16871 | 140 58 41.49517 | 156.07901 | | |
| BELLARINE (GPS Ecc) | -38 9 10.2192 | 144 36 38.9519 | 144.050 | 135.820 | 8.23 |
| | -38 9 4.83008 | 144 36 43.66724 | 138.95233 | | |
| BENAMBRA (South Base) | -37 0 42.4517 | 147 39 48.3021 | 781.107 | 770.857 | 10.25 |
| | -37 0 36.93211 | 147 39 52.80853 | 784.78046 | | |
| CANN | -37 38 54.1332 | 148 58 39.7215 | 536.494 | 529.884 | 6.61 |
| | -37 38 48.58960 | 148 58 44.20631 | 540.16496 | | |
| CHAPPLE | -38 39 49.9180 | 143 27 1.0664 | 557.002 | 548.252 | 8.75 |
| | -38 39 44.58801 | 143 27 5.87277 | 548.94666 | | |
| GREDGWIN SILO (Ecc A) | -35 58 22.1290 | 143 37 6.6990 | 152.370 | 146.050 | 6.32 |
| | -35 58 16.72815 | 143 37 11.36347 | 152.25840 | | |
| HOLEY HILL | -38 14 0.0832 | 146 56 19.4618 | 224.935 | 217.875 | 7.06 |
| | -38 13 54.61751 | 146 56 24.05754 | 223.42837 | | |
| IDA | -36 52 50.0031 | 144 42 22.1910 | 458.166 | 450.366 | 7.80 |
| | -36 52 44.57793 | 144 42 26.82486 | 457.20060 | | |
| JUNG | -36 36 50.4342 | 142 21 24.6300 | 159.504 | 151.884 | 7.62 |
| | -36 36 45.08562 | 142 21 29.37108 | 155.47370 | | |
| KOSCIUSKO (Pillar) | -36 27 26.5620 | 148 15 44.1212 | 2239.550 | 2229.480 | 10.07 |
| | -36 27 21.01840 | 148 15 48.56910 | 2246.799 | | |
| MATLOCK | -37 34 35.1824 | 146 11 21.0457 | 1382.952 | 1372.482 | 10.47 |
| | -37 34 29.72636 | 146 11 25.64978 | 1382.41449 | | |
| SAMARIA | -36 51 20.9080 | 146 3 39.9282 | 960.625 | 952.425 | 8.20 |
| | -36 51 15.43894 | 146 3 44.51169 | 962.27992 | | |
| TALGARNO | -36 5 6.6509 | 147 5 43.4265 | 652.012 | 644.942 | 7.07 |
| | -36 5 1.13281 | 147 5 47.92021 | 657.66084 | | |
| WEEJORT | -37 33 6.7333 | 143 5 0.5094 | 377.698 | 368.608 | 9.09 |
| | -37 33 1.37561 | 143 5 5.27304 | 372.41748 | | |

## OTHER STATIONS

| Name | Lat.(DMS) (WGS84) | Long.(DMS) (WGS84) | h(m) (WGS84) | N(m) (ANS) |
|---|---|---|---|---|
| BARHAM RESERVOIR | -35 37 28.80639 | 144 8 4.19019 | 110.70807 | 6.02 |
| BRUMBY | -36 14 34.24287 | 148 8 44.96182 | 428.19672 | 9.65 |
| BULLANGINYA | -35 47 51.11853 | 145 33 13.53147 | 171.85280 | 6.13 |
| COBBIN (P) | -36 26 17.20504 | 148 35 10.43841 | 1279.79583 | 10.60 |
| EDEN BREAK WATER(P) | -37 4 27.41972 | 149 54 28.48513 | 17.49208 | 5.42 |
| EUSTON RESERVOIR | -34 34 39.20794 | 142 44 39.51179 | 86.21749 | 4.59 |
| LAKE LITTRA | -33 55 45.92358 | 141 0 9.87804 | 34.59868 | 3.97 |
| LIANIDUCK (RM3 S) | -35 15 12.43957 | 142 53 42.75233 | 97.66481 | 5.44 |
| LOKA | -35 51 53.36338 | 147 4 0.63218 | 680.66866 | |
| MAJOR (RM3 BRASS) | -36 21 46.19794 | 145 41 46.13340 | 385.73656 | 7.07 |
| MOORONG (P) | -35 6 51.92944 | 147 18 15.55993 | 312.33860 | 6.49 |
| MT GAMBIER (7022) | -37 50 24.96113 | 140 45 21.62636 | 183.80783 | 6.93 |
| THIELE (SA) | -34 16 53.42635 | 140 53 38.80679 | 53.02736 | 3.98 |
| TOWER HILL (1862) | -38 19 16.71546 | 142 21 35.03472 | 94.52845 | 8.12 |
| COBRAM (TS 72313) | -35 55 8.18846 | 145 38 53.07056 | 153.92041 | 6.30 |
| WENTWORTH LOCK | -34 6 36.32961 | 141 54 13.68485 | 37.78353 | 3.83 |
| YELTA (SSM) | -34 7 37.54072 | 142 0 19.55781 | 61.80925 | 3.85 |